

# Zero-field superconducting phase transition obscured by finite-size effects in thick $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ films

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We report on the normal-superconducting phase transition in thick  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  films in zero magnetic field. We find significant finite-size effects at low currents even in our thickest films ( $d=3200$  Å). Using data at higher currents, we can unambiguously find  $T_c$  and  $z$ , and show  $z=2.1\pm 0.15$ , as expected for the three-dimensional XY model with diffusive dynamics. The crossover to two-dimensional behavior, seen by other researchers in thinner films ( $d\leq 500$  Å), obscures the three-dimensional transition in both zero field and the vortex-glass transition in field, leading to incorrect values of  $T_c$  (or  $T_g$ ),  $\nu$ , and  $z$ . The finite-size effects, usually ignored in thick films, are an explanation for the wide range of critical exponents found in the literature.

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Since the discovery of high-temperature superconductors and the realization that their higher critical temperatures and smaller coherence lengths create an experimentally accessible critical region,<sup>1</sup> researchers have looked at these superconductors in an effort to determine the model that governs the phase transition. Fisher, Fisher, and Huse<sup>2</sup> codified the scaling approach to the normal-superconducting ( $N-S$ ) phase transition and predicted the existence of an  $N-S$  phase transition in field, called the vortex-glass transition. This phase transition in field has been extensively studied using current-voltage ( $I-V$ ) isotherms, and although a consensus has emerged that a vortex-glass transition exists, there is little consensus in the values of the critical exponents  $\nu$  and  $z$ .<sup>3-5</sup> Moreover, some have claimed that scaling data collapse does not prove the existence of a phase transition<sup>6</sup> and that screening can create a nonzero resistance,<sup>7</sup> destroying the transition. Recent work has questioned the existence of a phase transition, showing that data collapse alone is too flexible, and proposing a criterion to determine whether or not a phase transition has occurred.<sup>8</sup>

In zero magnetic field the existence of an  $N-S$  phase transition is not in doubt. Very close to  $T_c$  ( $|T-T_c|\leq 2$  K),<sup>1</sup> the transition is expected to obey the three-dimensional (3D)-XY model, with  $\nu\approx 0.67$  and  $z=2.0$  for diffusive dynamics.<sup>2</sup> Although specific heat and penetration depth measurements have found mean-field values of  $\nu\approx 0.5$ ,<sup>9</sup> others have fit specific heat and penetration depth data using critical models with smaller residuals than for mean-field models,<sup>10</sup> and recent thermal expansivity data are more consistent with 3D-XY scaling ( $\nu\approx 0.67$ ).<sup>11</sup> Transport measurements can determine both  $\nu$  and  $z$ , but data in zero field are inconsistent: Researchers have found vortex-glass-like exponents ( $\nu=1.1$ ,  $z=8.3$ ) in small fields ( $<10$  mT) (Ref. 12); others find 3D-XY-like exponents when extrapolating to zero field from higher fields<sup>13</sup> and in crystals.<sup>14</sup>

Researchers have shown that, in thin films ( $d\leq 500$  Å), the fluctuation dynamics can cross over from  $D=3$  to  $D=2$ , and that this crossover occurs at a well-defined current density,  $J_{min}$ .<sup>2,20</sup> In this work we present a systematic study of  $J_{min}$  in films of different thicknesses. We find that even in our thickest film ( $d=3200$  Å) the crossover to  $D=2$  obscures the

phase transition, causing incorrect choices for  $T_c$ ,  $\nu$ , and  $z$ . However, at currents greater than  $J_{min}$ , we see behavior as predicted by scaling which gives reliable values for  $T_c$  and  $z$ .

We examined the zero-field  $I-V$  curves of  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  (YBCO) films deposited via pulsed laser deposition onto  $\text{SrTiO}_3(100)$  substrates. X-ray diffraction verified that our films are of predominately  $c$ -axis orientation, and ac susceptibility measurements showed transition widths  $\leq 0.25$  K.  $R(T)$  measurements (inset to Fig. 1) show  $T_c\approx 91.5$  K and transition widths of about 0.7 K. Atomic force microscope (AFM) and scanning electron microscope (SEM) images show featureless surfaces with a roughness of  $\approx 12$  nm. Our films also have a high critical current [ $J_c(77\text{ K})\approx 2\times 10^{10}$  A/m<sup>2</sup>]. These films are of similar or better quality than most YBCO films reported in the literature.

Our films were photolithographically patterned into four-probe bridges of varying widths (8–200  $\mu\text{m}$ ) and lengths (40–1000  $\mu\text{m}$ ) and etched with a dilute solution of phos-

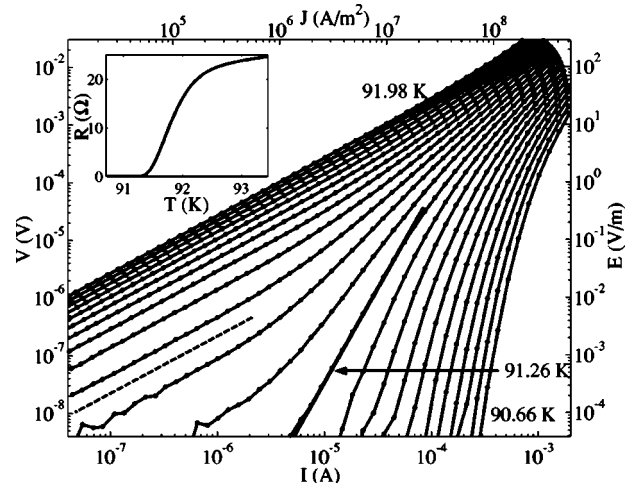


FIG. 1.  $I-V$  curves for a 2100 Å  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  film, with bridge dimensions  $20\times 100$   $\mu\text{m}^2$ , in zero magnetic field. Isotherms are separated by 60 mK. The dashed line indicates a slope of 1, or ohmic behavior. The error bars are smaller than the points. The inset is  $R(T)$  at 10  $\mu\text{A}$ .

phoric acid without noticeable degradation of  $R(T)$ . We surround our cryostat with  $\mu$ -metal shields to reduce the ambient field to  $2 \times 10^{-7}$  T inside. To reduce external noise, the cryostat is placed inside a screen room with low-pass T filters at the screen room wall and low-pass  $\pi$  filters at the cold end of the probe.

Figure 1 shows  $I$ - $V$  curves taken on a 2100 Å thick film on a bridge of dimensions  $20 \times 100 \mu\text{m}^2$ . Scaling predicts<sup>2</sup>

$$V\xi^{2+z-D}/I = \chi_{\pm}(I\xi^{D-1}/T), \quad (1)$$

where  $D$  is the dimension,  $z$  is the dynamic critical exponent,  $\xi$  is the coherence length, and  $\chi_{\pm}$  are the scaling functions for above and below the transition temperature  $T_c$ . Fluctuations are expected to have a typical size  $\xi$  that diverges near  $T_c$  as  $\xi \sim |T/T_c - 1|^{-\nu}$ , defining the static critical exponent  $\nu$ .

Above  $T_c$  at low currents, the  $I$ - $V$  curves are expected to be ohmic (represented in Fig. 1 as a dashed line with slope 1), whereas at higher currents the isotherms are expected to show nonlinear, power law behavior (slope greater than 1). Exactly at  $T_c$ , the coherence length diverges while the voltage remains finite, which is true only if  $V \propto I^{(z+1)/2}$  (for  $D=3$ ), i.e., a straight line on a log-log plot.<sup>2,3</sup> Conventionally,  $T_c$  is chosen as the first isotherm without an ohmic tail, the isotherm at 91.26 K in Fig. 1. Data at higher currents and voltages are typically excluded from fits because it is assumed that the system is being driven out of thermal equilibrium. The thick solid line at 91.26 K is a fit to a power law at lower voltages, and gives a dynamic exponent  $z=5.5$ , similar to exponents found elsewhere,<sup>12</sup> but clearly not the expected  $z=2$ .

We have suggested<sup>8</sup> that a better way to determine the critical isotherm is to examine the derivatives of  $\log E$  vs  $\log J$  isotherms. On such a graph, the critical isotherm would be obvious as a horizontal line with intercept  $(z+1)/2$ , separating isotherms with positive and negative slope (corresponding to concave up and down in Fig. 1). Our opposite concavity criterion<sup>8</sup> states that isotherms at equal temperatures away from  $T_c$  should show opposite concavity at the same current level.

The derivative plot for the  $I$ - $V$  curves in Fig. 1 is shown in Fig. 2. There is no isotherm that is horizontal over the entire range of currents, contrary to theoretical expectations. The opposite concavity criterion is also not satisfied, and isotherms below 91.44 K have unexpected behavior: they are concave down at higher currents before displaying ohmic behavior at lower currents. If we consider only the higher currents ( $I > 40 \mu\text{A}$ ), we can see behavior as predicted by scaling which also satisfies the opposite concavity criterion: the isotherm at 91.44 K is horizontal, lower isotherms are concave up, and higher isotherms are concave down. This allows an unambiguous choice for  $T_c$ , 91.44 K. If we fit the high-current data to a horizontal line, then  $z=2.1 \pm 0.15$ , which agrees with diffusive dynamics. Below we will justify analyzing only  $I > 40 \mu\text{A}$ , ignoring the low-current linear behavior in these  $I$ - $V$  curves (the “ohmic tails”).

To determine whether the ohmic tails are a bulk intrinsic effect, we patterned bridges of different widths on the same film. The inset to Fig. 2 shows the 91.26 K isotherm for three bridges on the same 2100 Å film from Figs. 1 and 2. Each

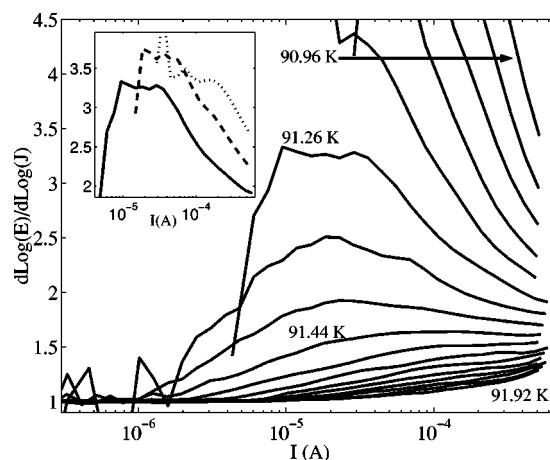


FIG. 2.  $d\log E/d\log J$  vs  $I$  for the  $I$ - $V$  curves from Fig. 1. Isotherms are separated by 60 mK. The conventional choice for  $T_c$ , 91.26 K, is clearly not a horizontal line. The opposite concavity criterion can be seen at higher currents ( $I > 40 \mu\text{A}$ ) about 91.44 K. The inset shows the 91.26 K isotherm for three bridge widths on the same film: 20  $\mu\text{m}$  (solid line), 50  $\mu\text{m}$  (dashed line), and a 100  $\mu\text{m}$  (dotted line), which do not agree as a function of  $I$ .

bridge was measured simultaneously to ensure identical temperatures. It is clear that the isotherms do not agree as a function of  $I$ . In Fig. 3, we plot  $d\log E/d\log J$  as a function of  $J$  rather than  $I$ . All three bridges have similar behavior in  $J$ , showing that we are measuring a bulk effect as opposed to an edge effect.<sup>15</sup> It is also clear that each isotherm turns over towards ohmic behavior at a certain applied current density rather than current. This is significant, because at an applied current density  $J$  one probes fluctuations of typical size<sup>2,20</sup>  $L_J = (ck_B T / \Phi_0 J)^{1/2}$ , where  $\Phi_0 = h/2e$  is the magnetic flux quantum and  $c$  is a constant expected to be of the same order as the YBCO anisotropy parameter,  $\gamma \approx 0.2$ . Thus, as  $J$  decreases,  $L_J$  increases and will eventually reach the thickness of the film. At this point a crossover to 2D behavior is expected, as the size of the fluctuations is limited along the  $c$ -axis. Thus, for a film of thickness  $d$ , there is a minimum

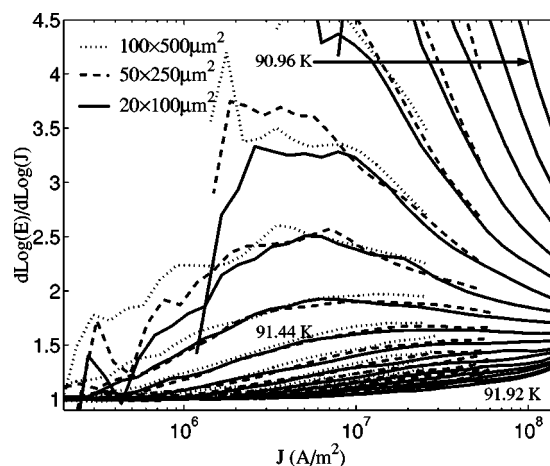


FIG. 3.  $d\log E/d\log J$  vs  $J$  for three bridges of different widths on the same 2100 Å film ( $20 \times 100 \mu\text{m}^2$ ,  $50 \times 250 \mu\text{m}^2$ ,  $100 \times 500 \mu\text{m}^2$ ). The crossover to ohmic behavior clearly depends on  $J$ .

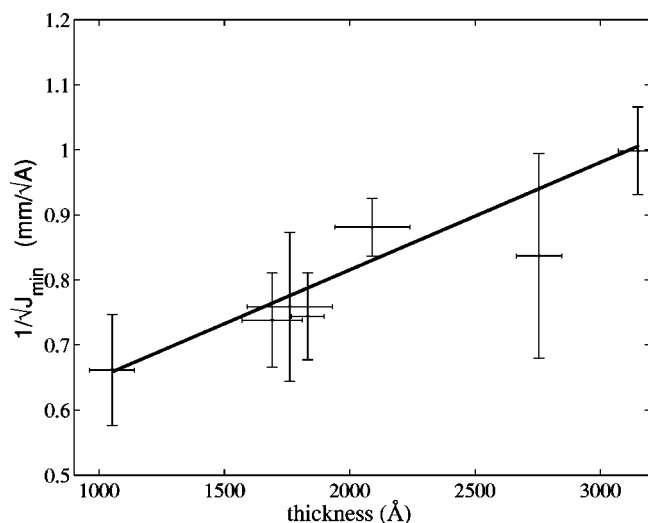


FIG. 4.  $1/\sqrt{J_{min}}$  vs thickness for eight different films. The solid line is a weighted least-squares linear fit to the data, with  $\chi^2 = 0.41$ . The slope of the line gives  $c = 0.60 \pm 0.17$ , of the same order as  $\gamma \approx 0.2$ , as expected.

current density, such that smaller current densities probe 2D fluctuations:

$$J_{min} = ck_B T / \Phi_0 d^2. \quad (2)$$

Because  $J_{min}$  does not depend on the exponents  $\nu$  and  $z$ , this minimum current density applies for both the vortex-glass transition and the transition in zero field.

We examined the ohmic tails generated in seven films of different thicknesses to determine how  $J_{min}$  varies as a function of thickness. We measured seven films with similar properties ( $T_c$  and  $\Delta T_c$ ) which varied in thickness from 950 Å to 3200 Å.<sup>16</sup> To choose  $I$ - $V$  curves to compare between films, we have taken the isotherm which, from high-current data, most seems like  $T_c$ , i.e., horizontal on the  $d \log E / d \log J$  vs  $J$  plot. For  $J_{min}$  we have chosen a similar criterion as Ref. 17, when  $d \log E / d \log J = 1.2$ .<sup>18</sup> If the ohmic tails are caused by finite-size effects, then we expect  $1/\sqrt{J_{min}}$  vs.  $d$  to be a line with slope  $(\Phi_0 / ck_B T)^{1/2}$ , which will give a value for the undetermined constant  $c$ . The temperature  $T$  of the different isotherms only varies from 91.4 K to 92.5 K, a total change of about 1%.

The results are plotted in Fig. 4. Each value for  $1/\sqrt{J_{min}}$  incorporates error in  $I_{min}$ , bridge width and thickness, leading to error bars of about  $\pm 22\%$ . Nonetheless, the trend is clear: as  $d$  increases,  $J_{min}$  decreases. The solid line in Fig. 4 is a weighted least-squares linear fit to the data with a reduced  $\chi^2$  value of  $\chi^2 = 0.41$ .<sup>19</sup> From the slope we determine  $c = 0.60 \pm 0.17$ , the same order of magnitude as  $\gamma \approx 0.2$ , as expected.

These finite-size effects have been seen by other researchers in thinner films. Dekker *et al.*<sup>17</sup> found  $z = 2.2 \pm 0.4$  from high-current data in a 500 Å thick YBCO film in zero field, and saw ohmic tails at low currents. Using an equation nearly identical to Eq. (2), Dekker *et al.* noted that the fluctuation size along the  $c$ -axis saturated at  $\approx 470$  Å, as expected.

Finite-size effects have also been seen in single crystals, both in a field and in zero field. Yeh *et al.*<sup>14</sup> found ohmic deviations from their data collapse at low currents and attributed them to finite-size effects. The length derived from Eq. (2) agreed well with the distance between twin boundaries, and they suggested these boundaries limited the size of the fluctuations. Although they found good agreement between theory and experiment, the deviations were determined after the data collapse. We have shown that  $I$ - $V$  curves can be made to scale with different choices of  $T_g$ ,  $\nu$ , and  $z$ ;<sup>8</sup> thus apparent agreement with scaling via a data collapse is not conclusive evidence that a phase transition occurs, or that one's choice of critical parameters is the correct one.

Wöltgens *et al.*<sup>20</sup> found deviations from 3D scaling which appeared as ohmic tails in films with  $d \leq 500$  Å, as compared to a 3000 Å film. Wöltgens *et al.* assume that the finite-size effects in thin films do not extend to the 3000 Å film because the  $I$ - $V$  curves for the 3000 Å film scale with typical vortex-glass exponents, despite the fact that Yeh *et al.* found finite-size effects in crystals, where the distance between twin boundaries were  $\approx 2$  μm, nearly an order of magnitude thicker than the thickest films of Wöltgens *et al.* Moreover, a simple data collapse is not conclusive evidence that the 3000 Å thick films are unaffected by the finite-size effects they see in thinner films. For  $T = 83$  K,  $J_{min} \approx 1 \times 10^6$  A/m<sup>2</sup>, and data below  $J_{min}$  were included in their analysis of the 3D transition, and included in the scaling collapse, possibly affecting their choice of  $T_g$ . This indicates that assuming a data collapse *a priori* and analyzing deviations from this collapse is not the correct method to determine finite-size effects.

Because the crossover to  $D=2$  can affect the choice for  $T_c$ , it is an explanation for the wide range of critical exponents found in the literature.  $I$ - $V$  curves are expected to be ohmic at low currents for  $T > T_c$  (or  $T_g$ ); thus it is possible to confuse ohmic tails generated by finite-size effects with ohmic tails generated by the 3D phase transition. This changes the conventional choice for  $T_c$  (or  $T_g$ ) [the first isotherm without an ohmic tail], and because the ohmic tails are used to determine  $\nu$  [as  $R \propto (T/T_c - 1)^{\nu(z-1)}$  at low currents], then values for  $\nu$  and  $z$  will also be affected. For example, Sawa *et al.*<sup>21</sup> scaled  $I$ - $V$  curves for films as thin as 180 Å and as thick as 10 000 Å in a 2 T applied field by systematically changing  $T_g$ ,  $\nu$ , and  $z$ , citing this as evidence for the need for an anisotropic 3D-XY model. We suggest that the crossover to  $D=2$ , occurring at different current densities, required them to vary  $\nu$  and  $z$  in order to scale their data.

Our results indicate that low-current ohmic tails are due to finite-size effects. Failure to account for this leads to significant underestimates of  $T_c$  (or  $T_g$ ) and incorrect values of  $\nu$  and  $z$ . We show in the derivative plot that the high-current data agree with the opposite concavity criterion and lead to unambiguous choices of  $T_c$  and  $z$ . Because the source of the low-current ohmic tails is in question, this leaves only the data collapse to find  $\nu$ , but using data collapse to find the critical exponents is perilous.<sup>8</sup> We can collapse the data using  $T_c$  and  $z$  found from the high-current data, which yields  $\nu \approx 1.2$ , similar to values found elsewhere,<sup>12,22</sup> but this value for  $\nu$  is clearly not 3D-XY, and other values of  $T_c$ ,  $\nu$ , and  $z$  can collapse the data also. It is also unclear how the ohmic tails affect data at higher currents, especially when  $T$  is far from  $T_c$  and the critical region is small.



It is also interesting to note that as  $d$  increases,  $I_{min}$  does not decrease without limit. Because  $I=J(wd)$ ,  $I_{min}=(w/d) \times (ck_B T/\Phi_0)$ , using Eq. (2). The smallest  $I_{min}$  can be is when  $w=d$ , or  $I_{min} \approx 1 \times 10^{-7}$  A ( $T=90$  K). Thus *any* applied current below  $0.1 \mu\text{A}$  will probe fluctuations limited by the thickness of the sample, independent of whether the sample is a thin film, thick film, or single crystal.

In conclusion, we have looked at YBCO microbridges of various widths (8–200  $\mu\text{m}$ ) in seven films of different thicknesses (950–3200  $\text{\AA}$ ) whose zero-field  $I$ - $V$  curves are consistent with low-current ohmic tails created by finite-size effects, even in the thickest films. In contrast, the behavior at currents greater than  $J_{min}$  ( $I > 40 \mu\text{A}$  in our film) agrees with the opposite concavity criterion as predicted by scaling, and gives the expected 3D-XY dynamic exponent of  $z = 2.1 \pm 0.15$ . Because finite-size effects are usually ignored in

thicker films, we suggest that the low-current ohmic tails thought to be the expected behavior for  $T > T_c$  are actually generated by finite-size effects at temperatures  $T > T_c$  and  $T < T_c$ . This effect will obscure the phase transition in all films, both in zero and nonzero magnetic field, leading to incorrect results for the critical exponents and temperatures.

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<sup>16</sup>Films thinner than 950  $\text{\AA}$  and thicker than 3500  $\text{\AA}$  tend to have a lower  $T_c$  and a broader  $\Delta T_c$ .

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<sup>19</sup>The line fit in Fig. 4 does not have a zero intercept, as one would expect from Eq. (2). This most likely results from a low choice

of  $J_{min}$ . The transition from a power law to ohmic occurs over more than a decade, and  $d\log E/d\log J=1.2$  is towards the lower end of this transition. However, the nonzero intercept does not significantly change the value of  $J_{min}$ .

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