

Normal-superconducting phase transition mimicked by current noise

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As a superconductor goes from the normal state into the superconducting state, the voltage versus current characteristics at low currents change from linear to nonlinear. We show theoretically and experimentally that the addition of current noise to nonlinear voltage versus current curves will create ohmic behavior. Ohmic response at low currents for temperatures below the critical temperature T_c mimics the phase transition and leads to incorrect values for T_c and the critical exponents ν and z . The ohmic response occurs at low currents, and will occur in both the zero-field transition and the vortex-glass transition. Our results indicate that the transition temperature and critical exponents extracted from the conventional scaling analysis are inaccurate if current noise is not filtered out. This is a possible explanation for the wide range of critical exponents found in the literature.

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The occurrence of a wide critical regime of the high-temperature superconductors¹—and the subsequent theories regarding the phase transition that occurs in this regime²—have led many researchers to look for critical behavior in the nonlinear voltage versus current (I - V) characteristics of these superconductors.³ This behavior has been studied in many materials in a variety of different conditions. The most widely researched material is $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (YBCO): thick films (thickness $d \approx 2500 \text{ \AA}$),⁴ thin films ($d < 1000 \text{ \AA}$),⁵ and bulk single crystals.⁶ YBCO has been measured both in a magnetic field (the vortex-glass or Bose-glass transition) and in zero field.⁷ Researchers have also investigated the vortex-glass transition in (to name but a few) $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$,⁸ $\text{Nd}_{1.85}\text{Ce}_{0.15}\text{CuO}_{4-\delta}$,⁹ and other more unusual superconductors,¹⁰ and critical behavior has even been reported in some low- T_c systems.¹¹ This large body of work has led to the general consensus that the vortex-glass transition exists, despite some arguments to the contrary.¹² However, there is a wide range of reported critical exponents ν and z from the experimental I - V curves. Our recent work has called into question the validity of the conventional scaling analysis,¹³ as we demonstrated multiple data collapses, each with its own set of critical parameters, using only one set of experimental data.

In this Rapid Communication, we discuss the underappreciated and invidious behavior of current noise when measuring nonlinear I - V curves. The normal-superconducting phase transition manifests itself at low currents as a change from ohmic behavior ($T > T_c$) to nonlinear behavior ($T < T_c$). We show, both theoretically and experimentally, that the addition of current noise to a device with an intrinsic nonlinear response will create an ohmic response at low currents. Thus, current noise will create ohmic behavior at low currents even for temperatures below T_c , and isotherms that are actually below T_c will appear to be above T_c . In this manner, current noise will mimic the phase transition and will lead to an underestimate of T_c , and incorrect values for ν and z —and in the worst case, the ohmic response due to noise will give the impression that the phase transition does not exist. This will

occur both in zero and nonzero field. Thus, different amounts of current noise (highly dependent on the experimental setup) will lead to different values for the critical exponents (expected to be universal). This effect, especially when combined with the flexibility inherent in scaling,¹³ is a possible explanation for the many different critical exponents reported in the literature.

To understand this effect more fully, we look at the underlying equations. When measuring the I - V curves of superconductors, we apply a dc current I_0 and measure the average voltage $\langle V \rangle$. Let us suppose that at some temperature T the sample has a response $V = f(I)$, where $f(I)$ can be nonlinear in current, and $f(I) = -f(-I)$, i.e., antisymmetric. Because any applied current will have noise (which may be shot noise, Johnson noise, $1/f$ noise, or noise from external sources such as the electronics), the measured voltage $\langle V \rangle$ will be given by¹⁴

$$\langle V \rangle = \int_{-\infty}^{\infty} f(I)P(I - I_0)dI, \quad (1)$$

where $P(I - I_0)$ is the probability distribution for the current, which is centered about the applied current I_0 . We assume $P(I - I_0)$ is symmetric about I_0 , as there is no preferred direction for current noise to flow. $P(I - I_0)$ has a width σ_I given by the variance of the probability distribution, $\sigma_I^2 = \int_{-\infty}^{\infty} (I - I_0)^2 P(I - I_0)dI$.

When $I_0 \gg \sigma_I$, the distribution is very narrow, and only values of $f(I)$ within a few σ_I will contribute to $\langle V \rangle$ in Eq. (1). We expand $f(I)$ in a Taylor series to find $f(I) = f(I_0) + f'(I_0)(I - I_0) + 1/2 f''(I_0)(I - I_0)^2 + \dots$. When inserted back into Eq. (1), due to symmetry, only the even terms in the expansion contribute, thus¹⁴

$$\langle V \rangle = f(I_0) + \frac{1}{2} f''(I_0) \sigma_I^2 + \dots, \quad (2)$$

and we see that

$$\langle V \rangle \approx f(I_0), \text{ for } I_0 \gg \sigma_I. \quad (3)$$

Thus, the finite width of $P(I-I_0)$ has no effect when the applied current I_0 is much larger than the noise current σ_I , and the measured voltage is independent of the noise.

The situation is markedly different when $I_0 \ll \sigma_I$. In this case, because I_0 is small, we will expand the probability distribution to first order in I_0 : $P(I-I_0) = P(I) + (-I_0) \partial P(I-I_0) / \partial I |_{I=0} + \mathcal{O}(I_0^2)$. When this distribution is inserted back into Eq. (1), we find that the first term does not contribute due to symmetry and thus, to first order in I_0 ,¹⁴

$$\langle V \rangle \approx I_0 R_{eff}, \text{ for } I_0 \ll \sigma_I, \quad (4)$$

where R_{eff} is an effective resistance given by

$$R_{eff} = - \int_{-\infty}^{\infty} f(I) \frac{\partial P(I)}{\partial I} dI. \quad (5)$$

This means that, if $I_0 \ll \sigma_I$, the measured voltage is *always* linear in the applied current, independent of the form of $f(I)$. Even strongly nonlinear I - V curves will appear ohmic at low currents.¹⁵ This occurs both above and below T_c , and will occur in zero field as well as in the vortex-glass transition.

This ohmic response at low currents is especially damaging because it can mimic the true ‘‘ohmic tails’’ expected for $T > T_c$ in a phase transition. For $T > T_c$, as $I \rightarrow 0$ it is predicted that (for $D=3$)²

$$\frac{V}{I} \sim \left(\frac{T - T_c}{T_c} \right)^{\nu(z-1)}, \quad (6)$$

where ν is a static critical exponent and z is the dynamic critical exponent. Thus, an ohmic tail generated by noise via Eq. (4) can be easily mistaken for the ohmic tail expected from the phase transition in Eq. (6), especially as they are both predicted to occur at low currents.

In general, $\langle V \rangle$ and R_{eff} are impossible to determine analytically because the function $f(I)$ is unknown. The form of $f(I)$ is known in two regions: the normal state and at T_c . In the normal state, the sample is a simple resistor, such that $V = IR_0$. At T_c , the voltage is expected to be a power law in current, such that $V = bI^a$, where the exponent a incorporates the dynamic exponent z [$a = (z+1)/2$ for $D=3$].²

We can determine $\langle V \rangle$ for these two cases. We assume a Gaussian form for $P(I)$, since we expect the noise fluctuations in the leads to be the result of the (almost) random motion of a huge number of electrons (stochastic motion), such that

$$P(I - I_0) = \frac{1}{\sigma_I \sqrt{2\pi}} e^{-(I - I_0)^2 / 2\sigma_I^2}. \quad (7)$$

We can then insert $f(I)$ and this form for $P(I-I_0)$ into Eq. (1) to find $\langle V \rangle$. When $V = f(I) = IR_0$ (in the normal state, or at low currents when $T > T_c$ in the critical regime), we find

$$\langle V \rangle = I_0 R_0, \text{ in the normal state,} \quad (8)$$

as expected for a simple resistor.¹⁶ On the other hand, at T_c when $V = f(I) = bI^a$, we find at low currents that the measured

voltage is linear in the applied current, $\langle V \rangle = I_0 R_{eff}$, where R_{eff} is given by

$$R_{eff} = b \sigma_I^{a-1} \sqrt{\frac{2^{a+1}}{\pi}} \cdot \Gamma\left(\frac{a}{2} + 1\right), \text{ for } T = T_c, \quad (9)$$

and Γ is the gamma function.

For a given experimental I - V curve which is nonlinear at high currents and ohmic at low currents, we can fit its high-current behavior to a power law to find a and b , and its low-current ohmic tail to find R_{eff} . If we assume the ohmic tail is entirely caused by noise, we can estimate the noise necessary to create the ohmic tail, as

$$\sigma_I = \left[\frac{R_{eff} \sqrt{\frac{\pi}{2^{a+1}}}}{b \Gamma\left(\frac{a}{2} + 1\right)} \right]^{\frac{1}{a-1}}. \quad (10)$$

We can compare this estimate with the noise as measured with a spectrum analyzer.¹⁷

We have examined the phase transition in zero field using current versus voltage (I - V) curves of YBCO films deposited via pulsed laser deposition onto SrTiO₃ (100) substrates. X-ray diffraction verified that our films are of predominately c -axis orientation, and ac susceptibility measurements showed transition widths ≤ 0.25 K. $R(T)$ measurements show $T_c \approx 91.5$ K and transition widths of about 0.7 K. Atomic force and scanning electron microscope images show featureless surfaces with a roughness of ≈ 12 nm. These films are of similar or better quality than most YBCO films reported in the literature.

We photolithographically patterned our films into four-probe bridges of width $8 \mu\text{m}$ and length $40 \mu\text{m}$ and etched them with a dilute solution of phosphoric acid without noticeable degradation of $R(T)$. We surround our cryostat with μ -metal shields to reduce the ambient field to 2×10^{-7} T, as measured with a calibrated Hall sensor. We routinely achieve temperature stability of better than 1 mK at 90 K. To reduce noise, our cryostat is placed inside a screened room and all connections to the apparatus are made using shielded triaxial cables.

We have experimented with several different filtering schemes. We use only passive filters, so as not to introduce noise from an active filter. Our typical filtering scheme, similar to others reported in the literature,⁶ uses low-pass π filters (insertion loss of 3 dB at 4 kHz) at the screen room wall. We have also used low-pass T filters (3 dB at 2 kHz) and double-T filters (3 dB at 2 kHz with a sharper cutoff) at the top of the probe. Additionally, we modified our probe to accept filters at the cold end. At 90 K, the 3-dB point of the low-pass T filters shifts upwards to 70 kHz. We also used cold copper-powder filters¹⁸ that have a measured insertion loss greater than 60 dB for frequencies greater than 5 GHz.

The prediction that noise creates ohmic tails is easily seen experimentally. We can dramatically increase the amount of noise in our system by removing the filters and leaving the

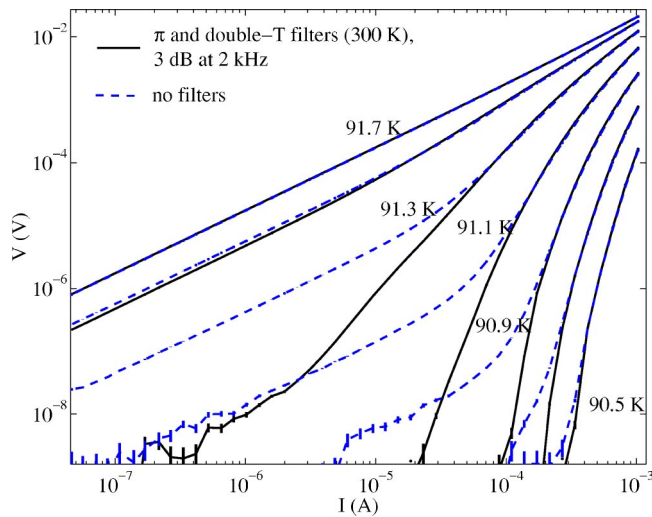


FIG. 1. Two sets of I - V curves for a 2100-Å-thick film with bridge dimensions $8 \times 40 \mu\text{m}^2$. The solid lines are isotherms taken with low-pass π filters (3 dB at 4 kHz) at the screened room wall and low-pass double-T filters (3 dB at 2 kHz) at the top of the probe. The dashed lines are isotherms taken without filtering. The isotherms are separated by 200 mK, and the error bars are shown (when larger than the lines). The highest-temperature isotherms (91.7 K) are ohmic, and fully in the normal state. In the transition region, we see (especially in the isotherms at 91.1 and 90.9 K) that noise creates ohmic tails in nonlinear signals.

door to our screened room open. We can then compare isotherms with and without filtering. Two sets of I - V curves for one sample are shown in Fig. 1.

In Fig. 1, the highest-temperature isotherm (91.7 K) is in the normal state, and has a slope of one (indicating ohmic behavior, $V \sim I$). In the transition region, the effect of noise is dramatically apparent. The filtered isotherms (solid lines) and the unfiltered isotherms (dashed lines) overlap at high currents, as predicted by Eq. (3), indicating that the additional noise has no effect. At lower currents, however, the unfiltered isotherms deviate and become ohmic (same slope as the isotherm at 91.7 K), as expected from Eq. (4). This effect is most noticeable in the isotherms at 91.1 and 90.9 K, where the nonlinear isotherms become ohmic at low currents when the filters are removed.

It is also easy to see how these ohmic tails due to noise could be mistaken for ohmic tails due to the three-dimensional (3D) phase transition. The ohmic tail due to noise at 90.5 K drops below the resolution of our voltmeter (1 nV), thus the unfiltered isotherm appears nonlinear. This transition from isotherms with an ohmic tail (91.1 and 90.9 K) to (apparently) nonlinear isotherms (90.5 K and below) is the same signature we expect from the phase transition. From the unfiltered isotherms alone, the conventional analysis of I - V curves would lead us to say that $T_c \approx 90.5$ K, despite the fact that the ohmic tails at 91.1 and 90.9 K are artifacts created by noise. Note also that the filtered and unfiltered isotherms are equally smooth. Once the noise reaches the sample, its response changes, thus the measured isotherm will appear smooth, regardless of how much noise is in the system.

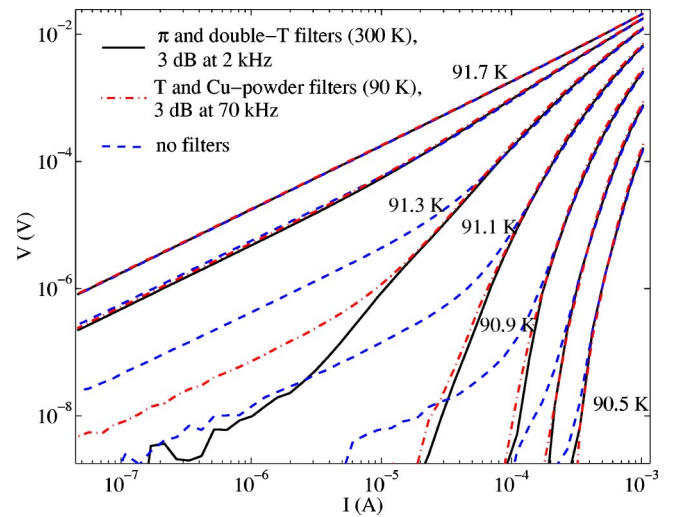


FIG. 2. Three sets of I - V curves for one sample. The solid and dashed lines are the same isotherms from Fig. 1. The dotted-dashed lines are isotherms taken with T filters and copper-powder filters at the cold end of the probe (3 dB at 70 kHz). The isotherms are separated by 200 mK and the error bars are suppressed for clarity. We can see that a 3-dB point of 70 kHz is not low enough to filter the isotherm completely.

In an attempt to further filter our leads, we added T filters and copper-powder filters¹⁸ to the cold end of our probe, very close to the sample. Isotherms taken with warm filtering at the screened room wall and the top of the probe (solid lines in Fig. 1) were identical to isotherms taken with filters at the screened room wall, top of the probe, and at the cold end of the probe. Thus, the addition of cold filters did not improve the data. From this we conclude that the Johnson noise created in the probe wiring is not significant.

It is instructive to consider the effect of cold filters alone versus warm filters alone. Because the 3-dB point of the T filters shifts to 70 kHz when cold, we can compare low-pass filters with different 3-dB points. Isotherms taken with all three filter configurations are shown in Fig. 2. From Fig. 2 it is obvious that a 3-dB point of 70 kHz is not low enough to filter the noise properly, although the difference between warm and cold filters is only obvious at 91.3 K.

This result leads us to wonder whether even a 3-dB point of 2 kHz is low enough to properly filter the data. Commercial passive filters with a 3-dB point lower than 2 kHz are hard to find, but we can resolve this question using another method. The environment connected to the sample generates a certain amount of current noise, but the I - V curves depend not on current but rather on current density. Therefore, if we test four-probe bridges of different widths, for a given amount of noise *current*, we can reduce the noise *current density* using wider bridges. We expect bridges of different widths to have similar J - E curves, where J is current density and E is electric field. However, if noise is still a problem, wider bridges should show different J - E curves. We have measured bridges of different widths,¹⁹ and have found that for typical filtering (3 dB at 2 kHz), the J - E curves for bridges of different widths (and thus different noise current densities) are identical, indicating that our low-pass π filters are sufficient filtering.

Additionally, we can take an isotherm and use Eq. (10) to estimate the amount of current noise required to create the ohmic tail. If we take the filtered isotherm at 91.3 K from Fig. 1, we find from the high currents $a \approx 1.55$ and $b \approx 10^{2.8} \text{ V/A}^a$. From the low currents, we find $R_{\text{eff}} \approx 0.7 \Omega$. We can plug this in to Eq. (10), and find $\sigma_I \approx 1.3 \mu\text{A}$, if the ohmic tail were caused by noise. We have measured the noise in our probe using a spectrum analyzer²⁰ and found $\sigma_I \leq 10 \text{ nA}$, far less than the estimate from Eq. (10), indicating that, with proper filtering, noise does not create the ohmic tails.²¹

Finally, it is interesting to note that we can change the resistance of the ohmic tail at 91.3 K by adding noise in Fig. 2. We know from Eq. (8) that adding noise to a linear I - V curve does **not** change the resistance. This result indicates that the underlying behavior at low currents of the 91.3 K isotherm *must* be nonlinear. The ohmic tail that occurs even in the filtered data must result from some other effect. In Ref. 19, we argue that this occurs due to the finite thickness of our films.

We have shown, theoretically and experimentally, that the addition of current noise can create ohmic behavior at low currents in nonlinear I - V curves. We have also shown that, in our experimental set up, passive low-pass π filters eliminated the effects of noise. However, without filters, it is easy to confuse ohmic tails generated by noise with ohmic tails expected from the phase transition, causing incorrect choices of T_c , ν , and z . These exponents are expected to be universal, although many different exponents are reported in the literature. Filtering schemes are rarely explicitly mentioned in the literature, and thus current noise may be a possible explanation for the lack of consensus regarding the exponents.

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³See Ref. 13 and references therein for a short list of the many papers using I - V curves to examine critical behavior.

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¹⁵Equation (4) is always true for any symmetric $P(I-I_0)$ and anti-symmetric $f(I)$, given that $I_0 \ll \sigma_I$.

¹⁶Again, Eq. (8) is true for any symmetric $P(I-I_0)$.

¹⁷Equations similar to Eq. (10) can be derived for other distributions (such as the Lorentz distribution).

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²⁰This is the measured noise for the typical filtering scheme (the solid lines in Figs. 1 and 2), and is independent of temperature. We did not measure the noise for the other filtering schemes.

²¹ σ_I does not change significantly for the Lorentz distribution, which is not surprising. We expect ohmic tails to occur when $I_0 \approx \sigma_I$. We see the ohmic tails occur at about $2 \mu\text{A}$ (at 91.3 K), thus we expect $\sigma_I \approx 2 \mu\text{A}$, independent of the exact form of $P(I-I_0)$.